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ABSTRACT

This document summarizes "Mathematics Counts," the report of the Committee of Inquiry into the Teaching of Mathematics in Schools in Great Britain, chaired by Dr. W. H. Cockcroft. It provides a digest of the report, paralleling the following chapters: (1) why teach mathematics? (2) the mathematical needs of adult life, (3) the mathematical needs of employment, (4) the mathematical needs of further and higher education, (5) mathematics in schools, (6) mathematics in the primary years and in the secondary years, (7) calculators and computers, (8) assessment and continuity, (9) examinations at 16+, (10) mathematics in the VIth form, (11) facilities for teaching mathematics, (12) the supply of suitable qualified mathematics teachers, (13) initial training courses, (14) in-service support for teachers, (15) some other matters, and (16) the way ahead. (MNS)

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A SUMMARY OF THE COCKCROFT REPORT "MATHEMATICS COUNTS"

by

members of the
ASSOCIATION OF TEACHERS
OF MATHEMATICS

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The Association was formed in 1952 by a small group of teachers of mathematics who believed that a reform in the teaching of mathematics required the development of new methods. Since that time the Association has gained a worldwide reputation for its methods of working and its achievements in the field of mathematical education.

Who Belongs To ATM?

The Association brings together people with a common interest in development work from all fields of education and includes in its membership teachers in primary and secondary schools, those involved in initial and in-service education of teachers (including students), lecturers in colleges and universities, inspectors and advisers. The current membership is around 5000 with a substantial proportion of overseas members.

Aims and activities

The Association works for changes in mathematical education which will relate it more closely to the powers and needs of children, and to the nature of mathematics as it is understood at the present time. Membership brings with it an open invitation to join one or more of the Association's working groups. These groups are formed in a variety of ways. There are local branch activities, an annual Easter conference which brings together people from this and other countries to work at issues in mathematical education over several days, working weekends at which more specialised activities or themes are pursued and many other activities which bring together members to work at current issues. Joining in with the Association's activities provides access to informal support networks for members with common interests.

Publications

The journal Mathematics Teaching, published quaterly, is free to members and contains articles on all levels of Mathematical education. The ATM supplement is also free to members and contains writing of a more provisional and controversial nature.

Several influential books have been written by ATM members and published by Cambridge University Press. The Association also publishes a variety of material arising out of the work of its members (see publications list).

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Introduction

The Committee of the Association of Teachers of Mathematics felt that it would be vital to make the Cockcroft report accessible. It arranged that a digest or summary should be available as soon as possible after the publication of the report.

It is emphasised that this summary does not replace the full report; rather it is hoped that more people will be encouraged to read the Cockcroft report in full.

We found the Cockcroft report readable and as a document responding both to educational and political problems, finely balanced between what it could say and what it could not.

It follows that to read between the lines the report itself must be studied. The outline we provide here is no more than a start.

There are many issues raised by the report that the Association will wish to work on and it is expected that a considered statement of priorities for work will become available shortly.

"Mathematics Counts" the Report of the Committee of Enquiry into the teaching of Mathematics in Schools under the Chairmanship of Dr W H Cockcroft. Available at HMSO price £5.75

Part 1

Chapter 1: Why Teach Mathematics?
The chapter outlines general reasons for teaching mathematics but emphasises particularly its communicating power through its concise and unambiguous nature; its ability to represent, to explain and to predict.
It draws attention to a fundamental paradox in that the very symbolism which contributes to its power often hinders many who try to learn it. Consequently "those who have been able to develop confidence with relative ease should not under-estimate the difficulties which many others experience ..."

Chapter 2: The mathematical needs of adult life
The trap of merely producing a "shopping list" of needs is avoided, after drawing on the experience of enquiries into adults' relationship to mathematics in their lives. A commissioned survey[1] demonstrates the extent of the fear, guilt and distaste for figures at all levels of society as well as the confidence. (The presumed need to be fast at computation and at the same time to avoid errors, leads many people to worry about ridicule, and to avoid mathematics when possible). About half

[1]

the people originally approached refused to be asked questions as "it was mathematics".
A section on numeracy rejects the notion (found in many submissions) that it merely describes the ability to do basic calculations. It should refer to being 'at home' with using numbers, and to having some confidence in understanding their use by others.

Chapter 3: The Mathematical Needs of Employment
This crucial chapter starts from the many public criticisms about standards of mathematics amongst school leavers. They state, after hearing a diversity of employers' views directly that "The overall picture ... is much more encouraging ... .. We have found little real dissatisfaction amongst employers with the mathematical capabilities of those whom they recruit from schools Further conclusions suggest '... a degree of over-expectation ... as to the level of attainment ... which some school leavers are able to reach'(57).
Paragraph (130) on courses in training centres and colleges of further education explicitly states that they can be unnecessarily taxing on craft apprentices.
A very helpful range of case study surveys has been undertaken[2,3] Among their significant findings, observations and comments are:

Range of mathematics required:
- calculations used by individual workers in different technical areas tend to involve non-standard methods, often learned on the job;
- there is ambivalence towards calculators with an increasing acceptance of them on the shop floor, but they are still viewed with suspicion by some managers and supervisors trained in logarithms and slide rules;
- manipulation of fractions as taught in school is rarely needed;
- algebra is rarely used; organised formulae with straight number substitutions are quite common;

but very important is the ability to estimate and finally to have 'a feel for measurement', which is the substance of the main implication for schools. This involves understanding the nature and purpose of measurement, the different methods and situations and the ability to interpret measurement in a variety of ways.

Employers' selection tests (91-95): The report is very critical of aspects of the procedures used in devising and setting tests which in different ways serve necessary purposes. Criticisms included: inappropriateness of test content to job requirement; higher standards put than are necessary; poorly produced tests with ambiguities; jargon terms and mathematically incorrect language; rigid schemes of marking with no allowance for crediting different interpretations where appropriate; and

commonly no access for candidates, or their schools, to the kind of test that they face. The chief recommendation leads to the need for improved liaison.

Liaison: There is a careful appraisal of the evidence of the increasing number of school/industry liaison activities in the country, but although there is a final plea for three-way communication between school, further education and industry, no firm stipulation as to its form is made. Warning is given against once-for-all laid down solutions, emphasising that only through continual inter-action will improvements be maintained.

A range of needs specific to identified areas of employment are discussed. A final comment on the future warns that the growth of automation will increasingly change differentially the mathematical demands at each level, which requires that adequate foundations are laid in school to meet this.

Chapter 4. The mathematical needs of further and higher education. This chapter summarises the factual mathematical requirements of courses undertaken at various levels. Courses in further education tend to be governed by regulations from different bodies. The difficulties of diversity in confidence and attainment of school-leavers can give rise to anxiety when set courses are offered in colleges of further education. The main finding in connection with higher education is the drop in the number of people with "double mathematics" at A-level taking Mathematics Honours degree courses in universities (80% of intake in 1973, 55% in 1979). An aspect of mathematical requirements for Professional bodies is that statistics is emphasised largely because few school leavers will have studied it in depth.

Part 2 Work in Schools

Chapter 5. Mathematics in school.
'A Review of Research in Mathematical Education'[4] was commissioned. Some consequences are discussed:

The attainment in mathematics of children in the whole age range is confirmed for 1979 as: 68% having an O level or CSE (all grades); 26% O level grade A,B or C; 5-6% A level; 0.5% started mathematics degree. It is shown that the boundaries of grades in English GCE/CSE are different from those in Mathematics. Although CSE grade 4 mathematics is found at the median, where according to CSE principles one expects it to be, English CSE Grade 4 is lower, between the 60th and 70th percentile. Consequently many pupils will obtain significantly higher grades in CSE English than in Mathematics (197).

Attitudes towards mathematics are discussed in general, with respect to parents, and in terms of differences between boys and girls. Comment is made on the need to provide Welsh medium resources. Problems of linguistic impact on mathematics learning are pointed out in situations where English is not the first language.
Under Teaching and Learning of mathematics, the variation of speed, attainment and rates of learning is discussed mainly with reference to possible strategies to be adopted when these variations are manifest. It is asserted that mathematics is a difficult subject to teach and to learn. [228]

Understanding refers to the need to develop adequate theory for a complex situation. The distinction made between 'relational' and 'instrumental' understanding is seen as a useful, but limited, beginning. The section headed memory discusses the useful concepts of 'short term' or 'working' memory and 'long term' memory, showing that different work is to be undertaken where one or other of these is involved. Long term memory is effective when subject matter is assimilated as part of a network of associated items. Rote learning is related to the 'long' and 'short' term memory discussion. A distinction is made between 'fluent' performance where routines have been subordinated to the ability to think and use, and 'mechanical' performance, with little use beyond short term needs.

Teaching methods [240] involves the need to pay attention to three distinct elements - facts and skills, conceptual structures, and general strategies and appreciation - and should be organised accordingly. Classroom practice is seen as involving six kinds of activity [245]:-
i) exposition by the teacher; ii) discussion between teacher and pupils and between pupils themselves; iii) appropriate practical work; iv) consolidation and practice; v) problem solving, including applications; vi) investigational work.

Chapter 6 Mathematics in the Primary Years
(also drawn from chapters 5 & 7)
Most of the recommendations about what mathematics to teach in the primary years, and when and how to teach it, are based on the notion of a child who successfully engages in a variety of mathematical tasks. Naturally the criteria for success will vary according to the kind of task in question. Taken out of context, these criteria include:
- The child can apply what he knows to a new situation.[e.g.see 288,301,321]
- He can estimate the likely outcome of a measurement or calculation. [e.g. see 259-261,291,299]

- He has made some mathematics his own, and can use what he knows to illuminate the task on which he is engaged [329].
- He has begun to develop some of the skills of the mathematician in his handling of mathematical tasks. [351,346]
- He can act as a semi-autonomous problem solver. [321-4,347,369]
- He positively enjoys what he is doing. [345-7]
- He understands what he is doing [e.g.289,298,300,307,308,316, 334,336,343,347,349,369,370]. Understanding is described as "an internal state of mind that has to be achieved individually by each pupil, and cannot be observed directly by the teacher". Ability to solve a particular problem does not of itself indicate understanding of the relevant concept; a better indication can be given by discussion [232].

In a number of the classrooms the Committee visited children were in fact engaged successfully in a variety of appropriate mathematical tasks [347]. Furthermore, much of the evidence they received was supportive of the work of Primary schools [265]. However, in chapter 17 the Committee state that in general "the mathematical education which many young pupils are receiving is not satisfactory" and "major changes are essential" [500]. In some of the primary schools they visited, they found that the mathematics teaching was over formal; in a number of cases this arid approach to mathematics was in sharp contrast to the "lively and supportive" general atmosphere [347]. The primary chapter should be read with these general observations in mind.

The criteria listed above provide the basis for the Committee's comments and recommendations about the primary mathematics curriculum. In general, a piece of mathematics should only be pursued as far as children can understand it; [e.g. the study of fractions for most children should be confined to work with equivalent fractions, and to halving and quartering in practical situations [300]).

The broadening of the curriculum [286] to include measurement, shape and space, graphs, elementary logic, and problems related to everyday life [291-7], has been associated with an increase in the numbers of children who engage successfully with some mathematics [288]. The results of a 'back to basics' approach would produce children no more able to do calculations and less able to apply what they had learnt to practical problems [278].

The two central problems which face all teachers of mathematics are: the wide range of attainment in mathematics to be found in any given year group of children [340-4]; combined with the great difficulty of teaching the subject [228-9]. (Research suggests that around 9 to 11 there is already a 7 year spread of mathematics attainment in any given year group).

Despite this, every child must be helped to engage successfully, with enjoyment and understanding, in some mathematics. So the challenge for teachers is to match the mathematical task to the individual child. He must therefore be enabled to proceed at a pace appropriate to his grasp of the task in question.
In many situations this will mean not moving too quickly (316); as for example when children first meet mathematical symbolism at the age of 5(304); and when they are gaining practical experience of working on everyday problems(296).

The most able children should be given more mathematically challenging tasks, but secondary school texts will not do for this. They should not be left to their own devices; they need the teacher's support and encouragement (330-3). Low attainers should be individually diagnosed and helped, and the least able children need simple practical work related to everyday life. (334-8)

The Committee recommend a number of teaching methods which in their view help teachers to bring about a successful engagement between the individual child and the mathematical task (347,805). The methods they recommend include:-

- The teacher and the children (often in ability groups) should discuss both new mathematical activities and their outcomes (319). There should also be whole class discussions, and some problems should be posed with this in mind (317).
- Each new mathematical topic should be presented as a series of "practical and intuitive" activities. "Practical work" is given a variety of meanings, including: using apparatus (structural or measuring); working with and talking about problems without writing anything down (325-7). In general, the move to symbolise, formalise or generalise should follow practical work.
- There should be a wide variety of equipment readily available for children to use for these practical activities (608-11).
- The teacher should use textbooks or commercial workcards selectively and flexibly. New topics should be introduced through activity and discussion, for children can seldom learn a new piece of mathematics simply by reading about it. Textbooks and workcards are best used to provide a framework of ideas and backup material (312-20).
- The work should be carefully structured (e.g. 286,287,292,296, 298,299,305,etc).
- Wherever the mathematical task allows, children should be encouraged to act as semi-autonomous problem-solvers (321-4).
- The teacher's methods should reflect awareness of the need to make doing mathematics a positive experience for the child (345,347,369,370).

The Committee make further special recommendations about learning to calculate.
There should be more 'mental mathematics' throughout the primary years [315-6].
All children should develop mental methods of calculation. These are likely to differ from written methods that are taught. The differing strategies used by children to do mental calculations should be respected.
The ability to calculate effectively on paper or mentally requires understanding of place value and of the meaning of the four main operations. It also requires the secure and rapid recall of basic addition and multiplication facts. Many children cannot be expected to master these requirements by the age of 11 [316,298,264].
The ability to calculate is fostered by a teacher who adopts a flexible, varied and imaginative approach to work with numbers and the operations on them, both in the way problems are presented and in the language used to present them [306-10]; e.g. work on large numbers and on decimals should be used to foster the ability of the child to estimate [299].
In many schools children currently spend too much time in unproductive calculating practice [302].

School organisation: Staff should be encouraged to take a positive attitude to teaching mathematics; they do so in schools in which the teaching of mathematics was most obviously successful [347]. Recommended ways of organising include:
- involve staff in the planning and review of the mathematics that is taught, and how it is taught [363-4]
- Enable the stronger mathematics teachers to offer support to the weaker ones, perhaps by team teaching [351,355]
- Ensure sufficient, accessible resources for teachers and for children
- Avoid vertical grouping where possible, in order to reduce the range of attainment with which the teacher has to cope [349]. (It can be helpful to set children across several classes, particularly for the most able children [350])
- Appoint a mathematics Co-ordinator to plan, co-ordinate and monitor the mathematics work of the school. His main duties include: to prepare a scheme of work in consultation with the Head and staff; to provide guidance and support e.g. through meetings; to work alongside individual colleagues, - especially probationers, teachers new to the school, supply teachers, and teachers in training; to offer special help to any teacher who lacks confidence with mathematics; to help colleagues with children who have learning difficulties; to arrange school-based in-service; to be responsible for teaching resources; to liaise with other schools and with advisers; to keep in touch with current developments in mathematics education [354-6].

Where necessary there should be additional salary increments made available for a mathematics specialist [358,662].

LEA guidlines can be helpful if accompanied by in-service work and revised regularly [359-62]

Calculators (see also chapter 7(1)) are a necessity and a positive help in the primary classroom [384-8]. Research evidence strongly suggests that the use of calculators has no adverse effect on basic computational skill [377]. Calculators should be used to develop childrens understanding of fundamental mathematical principles [392]. Their use will affect the primary curriculum in a variety of ways: they will enable children to tackle a further range of investigations and of real-life problems; in both cases the problems will no longer have to be "fixed" to avoid difficult calculations [386]. Low attainers can by-pass the need for computational skill, always provided that they concentrate instead on how to decide which is the appropriate operation for any given problem [328].
More development of in-service work and of classroom materials are urgent priorities [388].

Computers: There is relatively little software available which can be used to assist the learning of primary mathematics and much of this is of poor quality or "gimmicky". Funds should be provided for more programs for primary children that encourage problem solving and logical thinking in a mathematical context [408-9].

Mathematics in the Secondary Years (drawn from Chapters 5, 9, 7)

The difficulty of teaching mathematics during this period is seen as largely due to 'the seven year difference' in mathematical understanding and attainment that exists among 11 year olds [228], the very different learning speeds present in any class (however composed) [436] and the need for sufficient prior understanding to support successful learning of a new topic [437]. To these inherent difficulties is added a 'vicious circle' that has developed within the examination provision which greatly disadvantages those whose attainment is average or below. "Up to 80% of pupils in secondary schools are following courses leading to examinations whose syllabuses are comparable in extent and conceptual difficulty with those which twenty years ago were followed by only about 25% of pupils" [442]. One result of this is that a candidate of just below average attainment will achieve only 20% success at CSE papers [444] so that "those whose attainment is limited are liable to experience continuing and dispiriting failure" [437]. This cannot be educationally

desirable nor can the trivialising of topics and questions which teachers and examiners feel forced to undertake for these pupils' sake [445]. It is not surprising that the feelings of inadequacy and failure generated lead to the disenchantment with mathematics confirmed by the Bath and Nottingham Studies [200][2,3]. Candidates who get a grade E at 'O'-level are similarly disadvantaged and it is regretted that some employers find such a qualification more acceptable than a 2 or 3 at CSE [446].

From this awareness of difficulties, they propose that the teaching syllabus should be constructed "from the bottom upwards" [450], by first considering what topics children of below average and average attainment can tackle; and then adding topics and increasing depth of treatment when forming syllabuses for higher attaining pupils [452]. ('Syllabus' is defined as a list of mathematical topics, while 'Curriculum' contains both what is taught and how it is taught [438]). They believe "it should be a fundamental principle that no topic should be included unless it can be developed sufficiently for it to be applied in ways which the pupils can understand" [451].

They see this approach as leading to a "differentiated curriculum" [452] which combines the common experience of a foundation list of mathematical topics for nearly all pupils with differing additional content, depth and assessment to try to ensure both that pupils are not required to tackle work which is inappropriate to their level of attainment and that those who are capable of going a long way are enabled to do so (They provide such a list, covering aspects of number, money, percentages, use of calculator, time, measurement, graphs, spatial concepts, ratio & proportion and statistical ideas [458]). There should be similar underlying approaches to teaching and mathematics should be presented and taught in a context in which it will be applied to solving a variety of problems. Problem solving, logical deduction, abstraction, generalisation, conjecture and testing should play a part in the work of all pupils and worthwhile skills should be practiced in order to enable their use in problem solving and not just for their own sake [456]. The differentiated curriculum should not limit the range of work attempted by higher attaining pupils since if pupils follow a course whose content is better matched to their level of attainment and rate of learning the resulting greater confidence and mastery should lead to an overall raising of standards [475].

During the early secondary years the examination syllabuses should not be allowed to exert too much influence with the curriculum differentiating slowly since differing needs can be met by varying approaches and depths within the same topics.

They make detailed consideration of what the foundation list and differentiated curriculum imply for lower attaining pupils (459-66), pupils whose attainment is very low (467-9), pupils for whom CSE and O-Level are intended (470-3), higher attaining pupils (474-81) and pupils whose attainment is very high (482).
One particular proposal is for an 'Extra Mathematics' paper at O-level for the 5 - 10% who could study it alongside and as an extension of O-level mathematics without extra time being allocated.
For all who take exams, the teaching syllabus should give the examination syllabus coherence by extending beyond it (473).

The committee turn next to matters of liaison (483-5) over 'Mathematics across the Curriculum', where the main need is to raise the level of awareness among teachers, particularly regarding the part which mathematics can play in presenting information with clarity and economy, and time allocation, (486-90) where they see as appropriate between 1/7 and 1/8 of the teaching week suitably distributed through the week and at different times during the day, with the number of double and single periods suited to the methods of working and to the mathematics curriculum (491). (Practical and investigational work is likely to need double periods (466)). They consider the organisation of teaching groups (491) with sections on setted groups (493), mixed ability groups (495) and individual learning schemes (489), focussing on the strengths and problems which each working pattern entails. Under Deployment of teaching staff they indicate strongly that teachers of other subjects should not teach mathematics to lower attaining pupils, but instead take pupils of average or above average attainment and be supported in this by the head of department, possibly even by shared teaching (502-6).

Heads of Department have a crucial role to play (507-17) and a wide range of responsibilities and certain of their duties can only be performed while teaching is in progress and the necessary time needs to be provided within the timetable. (A list is given (508), and includes planning, organisation, monitoring, liaison and professional development aspects.) They also need to keep up-to-date with developments through in-service training, and through awareness of professional journals. Their responsibilities and those "at all levels within the educational system ... should be properly defined ... and that responsibility should be linked with authority, accountability and assessment" (517).

Chapter 7 Calculators and Computers

The effects of the increasing availability of low cost calculators surface throughout the report and are also separately considered. The Committee's positive attitude to them is shown by their inclusion within the foundation list for secondary pupils and by their views on written and mental calculation generally. Their use in employment and adult life combined with positive research evidence lead the committee to believe that calculators can be used to assist and improve the teaching of mathematics in the classroom [374]. There is strong evidence from all the research studies that "the use of calculators has not produced any adverse effect on basic computational ability" [377]. They also stress that the user has the same need for mathematical understanding.
Consequently they recommend that:-
Some calculators should be available in all primary classrooms, with associated development work and in-service provision being needed [384].
All secondary pupils should be taught and allowed to use calculators, which should replace logarithm tables and, in time, trigonometrical tables [389-91].
Examination boards should design their syllabuses and examinations on the assumption that all candidates will have access to a calculator by 1985 and all secondary schools should be stocked with enough calculators to make this possible by then [394-5]. Such use of calculators should bring changes of content, order and emphasis. When used as checking devices, the feedback to pupils is immediate and 'neutral' [374].
Investigations can be assisted or suggested by the use of calculators, which enable large or awkward numbers to be handled easily. Decimals will assume greater importance in relation to fractions and time spent on practicing written calculation can be saved e.g. it is not profitable for pupils to spend time practising the traditional method of setting out long division on paper. [390]

Computer Studies [396-401] should not be staffed at the expense of good mathematics teaching. An estimated 600 good mathematics teachers have been 'lost' in this way. Ideally it should exist within a separate department. Unless there are separate LEA advisers, mathematics advisers will not be able to give sufficient support either to mathematics teachers or to microcomputer development work.

Microcomputers in Mathematics [402-413]: The Micro-electronics in Education Project is welcomed. Their materials will require a very extensive programme of support and in-service [406]. Hardware will continue to fall in price. There is

limited software at present, and only some of it is good, so more needs to be produced, at a cost in money, effort and time [407]. Development work is needed in (a) making programs for mathematical activities which will encourage problem solving [409]; (b) realising the potentiality of high resolution graphical display [411]; (c) making programs which can contribute to the mainstream mathematical work [413].

Chapter 8 Assessment and Continuity

Assessment is an essential part of the work of all teachers and needs to be carried out in a variety of ways, (e.g. marking, brief conversation, glance at work in progress) and for a variety of purposes [414]. Marking should be diagnostic, supportive and enable awareness of types of mistakes [415]. It is not necessary to mark everything, but problem-solving needs teacher's approval, while routine exercises could be self-marked to the benefit of pupils [417].
Some written record should augment the wealth of information all teachers carry in their heads [418]. It should record those qualities which can only be assessed by the professional judgement of the teacher [419].
Testing should never be an end in itself, but should inform future action [420]. There is a difference between 'learning for the test' and 'learning for future use' which tests may not discern. "No one has ever grown taller as a result of being measured" [421]. If LEAs use standardised tests they should be capable of diagnostic use, of identifying good practice and of identifying support needs [422]. When used to assess transfer they should be part of a wider assessment, since it is important to realise that they measure only some aspects of mathematical attainment [423].
Mathematics co-ordinators, heads of department and head teachers need to evaluate the work of groups of classes and of the school as a whole [424].
The work of the APU is potentially very valuable to all concerned with the teaching of mathematics. There should be an overall appraisal of the implications of its work. In particular all teachers should be made aware of the practical testing aspect [425].

Continuity requires that discussion take place between those who teach in primary and secondary schools in the same area in an atmosphere of mutual professional respect. All need to acquaint themselves with the methods and materials in use [430].
When tests are used, they should be conducted before transfer rather than during the first few unsettling days at secondary school [432].
Secondary schools should take notice of records received from

contributory schools but not to pre-judge pupils for whom a 'fresh start' may be beneficial [433]. Ignoring such records can lead to able pupils being given work which is not challenging, which can sour attitudes [434].
Continuity problems can also arise when pupils transfer to a sixth form course, particulary at another establishment [435].

Chapter 10 Examinations at 16+

The chapter is in two parts dealing in the first place with examinations intended for pupils whose attainment is within the top 60 - 65% , and in the second, with the evidence of achievements in mathematics of lower attaining pupils.

Recognising how deeply examinations affect the way in which mathematics is taught [518], the committee assumes two fundamental principles which should govern the establishment of examinations. Accepting that examinations should not impose inappropriate constraints on work in secondary schools, they believe that the methods of assessment should enable candidates to demonstrate what they do know rather than what they do not know and that they should not undermine the confidence of those who attempt them.

Welcoming the proposed single system of examination at 16+ [520] they suggest that combinations of papers can be taken so that there is one group focussing on Grade 2, another at Grade 4 and another at Grade 6 [523]. (These compare with O-level Grade B, CSE Grades 2 and 4). At each of these foci a spread of grades would be possible and different ways of taking a collection of papers of an appropriate kind at each focus are proposed [524-8]. This confirms the argument for a differentiated curriculum where the same set of papers cannot be suitable for all pupils [527]. A paper taken by all candidates could not accord with the two fundamental principles mentioned above unless it was suitable to be included in those focussed at grade 6 [528]. The recommendation not to offer alternative 'traditional' and 'modern' papers confirms the statement in Chapter 5 that such a distinction is now invidious [529].

In a sequence of important paragraphs [531-6] the committee outlines the necessity for an element of classroom based teacher assessment to be included in any examination of mathematics. This follows as a consequence of identifying the necessity for pupils to undertake practical work, problem solving and investigations; which itself is consequent on the general principle that assessment at 16+ should reflect as many aspects of mathematics as possible. They are realistic about the present

situation in recognising that the widely used written examinations have encouraged teachers to limit the work they do to that which is directly related to the type of question set in the examination. They foresee a period of development and training before their recommendation can be fully implemented. Nevertheless some mathematics teachers are already accustomed to assessing their pupils for public examinations.

Evidence of achievement in mathematics of lower-attaining pupils [537-58]

Concern is expressed that though an increasing number of pupils not in the top 60 - 65% attainment band are seeking qualifications, it is questionable whether they should be examined in the form appropriate to higher attaining pupils. Also an increase in the percentage taking the examination could lead to distortion of the Grade 6 focussed papers, in trying fruitlessly to satisfy the criterion of appropriateness for lower attaining pupils [539-40].

Comment is made on schemes of assessment already in use. They vary from tests of calculating skills given at Easter before leaving to profiles such as 'SLAPONS' [542]. Though a few cover a reasonably wide range of mathematical topics, most concentrate wholly or in great part on testing computational skills. "We find this a cause for concern because we believe that the use of such tests encourages teaching of the kind which attracted adverse comment in the report of the National Secondary Survey" [543]. Appealing to the principles laid down in [521] a recommendation is advanced for a set of graded criterion-referenced tests constructed so that they are positively related to possible progress from 14 - 16 [545-553]. This is discussed extensively and a final recommendation is made that a study be commissioned to consider the possibility of devising a means to provide evidence of achievement in mathematics of low-achieving pupils which will support and not conflict with the provision of suitable mathematics courses.

CHAPTER 11 Mathematics in the VIth form

The committee concentrates mainly on some matters concerned with A level and, more briefly, on the proposed I-level and sub I-level courses.

45 - 50% of those studying A level go to university in 1978. Although others went into other forms of higher education, the fact that A level is a 'stopping point' for many raises many questions of appropriateness [559-60]. The largest single 'user' of university entrants with A level mathematics are courses in engineering and technology

In a section on the teaching of A Level, there is mild criticism of the danger of dependence on exposition and a passive style of teaching [561]. It is hinted that wider activities should counter the aridity which this style of teaching tends to produce. These wider activities should include practical applications, modelling and topics from the history of mathematics [562]. More use should be made of books and more help given to wide study skills [562]. The more able in particular should be encouraged in appropriate independent study beyond the confines of a single subject course.

A single subject A level should not consist of pure mathematics only [565]. There is a need for a balanced course and though the claims of mechanics are high so are those of probability and statistics [566]. Curriculum studies are needed in these matters so that rather less content in both these areas are contained in the single subject papers [567-569].

The diversity of students' future interests makes it impossible to recommend one area rather than another [570]. An argument is put that as the main pressure for mechanics comes from those in Higher Education responsible for engineering, and as 76% of the entrants for these courses have physics A level, then that is where the mechanics should be studied [572]. There is no evidence that students from one type of A level course have performed consistently better or worse than students from another.

The committee rejects the way in which recommendations for core syllabuses clearly leads to overloading and does not wish to see any 'core' occupying more than 40% of the total syllabus. It expects that arrangements for continual review of the 'core' will be established [576-578].

There is a discussion of the so-called proliferation of A level syllabuses and a comment that the number of papers is as much to do with the differing ways in which a variety of students can be satisfied while studying roughly the same syllabuses [579-581]. Once the modern and traditonal distinction is dropped then the number of syllabuses will fall [582]. Nevertheless it is essential that different possibilities are available in order to cope with different styles of teaching, and to make possible curriculum development [583]. No strong argument exists for arbitrary reduction of syllabuses except on the grounds of duplication [584]. It is recommended that monitoring of A level syllabuses is valuable and should continue.

Double mathematics is felt to be increasingly uneconomic in terms of over-specialisation, and use of teaching time [586].

Able students can benefit by working on their own (587). Institutions of higher education should be encouraged to drop pressure for students to take double mathematics (586). When it is taken it should be of the form mathematics and Further Mathematics, and not Pure and Applied (589).

Formula sheets have been criticised as being contributory to students not possessing manipulatory skills and not being able to recall frequently used single formulae (590). The committee support the arguments of examination boards who provide formula sheets for their A level examinations (591). In emphasising that need for quick re-call still exists, the committee recommends their continued provision at A level and that formulae related to 'common core' should appear on the formula sheets for all boards (593-594).

Other courses: I-Level, sub-I-Level and O-Level/CSE
At I-Level broad courses rather than those involving heavy manipulations are preferred. Statistics might be better treated here (595-597). Criteria for sub I-Level work should be found in chapter 3 on "Mathematical Needs of Employment". There is a warning against arid retakes of O-Level/CSE. Students often benefit from a more adult approach.

CHAPTER 12 Facilities for Teaching Mathematics

Primary years
Each class needs facilties and apparatus for practical work, but no special accommodation is required (600).
Much of the necessary equipment for practical work is neither elaborate nor expensive. There should be enough made readily accessible (608).
Young children also need to use simple structural apparatus of various types (611).

Thought should be given to library provision of general interest mathematics books, especially for younger pupils, and these need prominent display (614).

Secondary years
Mathematics should be taught in suitably equipped specialist rooms. Their provision makes it easier for good practice to develop (304). It would be preferable for such rooms to be grouped together and to have good facilities and equipment (605). Such accommodation provides opportunities for co-operation and support (606), for more effective use of equipment and resources, and for more flexible arrangements generally. Specialist rooms can also house a small mathematics library (615).

Very much more practical material needs to be readily available in secondary classrooms [612]. Up to date displays of material and of pupils work are valuable [613]. Libraries should place general interest mathematics books together.

Primary and Secondary years

There should be reference material for teachers including publications of the professional mathematical associations, DES, HMI and Schools Council teachers' guides and a selection of text books [616].

It is essential that sufficient money should be made available to maintain adequate stocks of books and equipment and that flexibility exists in the way in which money allocated for mathematics can be used [617].

CHAPTER 13 The supply of suitably qualified mathematics teachers

In order to place a measure upon the shortage of such teachers, the committee defined 4 levels of qualification: 'good', 'acceptable', 'weak' and 'nil', with the former two being 'suitable and the latter two 'unsuitable'. Using the most recent statistics available[5] they found that almost 2/5 of all secondary mathematics teaching was in 'unsuitable' hands [630]. So was more than 3/5 of all secondary-age mathematics teaching in middle schools [626]. This is equivalent to a shortfall of around 9,500 teachers, over 5,000 of whom have 'nil' qualification [619,A13].

Given that mathematics is especially vulnerable to weak teaching [619] and that re-deploying teachers could make minimal difference only [633], the Committee propose three ways in which the supply of suitable teachers could be increased [637]
- try to increase suitably qualified entrants to teaching
- try to ensure that suitable teachers remain in teaching
- try to improve the quality of some under-qualified teachers who are now teaching mathematics. (In relation to this the DES Retraining and Further Training Courses are making a welcome contribution)

The serious, steady decline in the proportion of mathematics graduates entering teaching[6] could in part be due to pay and prospects [638]. For the ages 26-35, mathematics teachers averaged £1,000 - £2,000 p.a. less than similarly qualified contemporaries in industry, commerce and government [655]. In addition graduates perceived discipline problems and low morale in schools [639]. The Committee seek central and local government response in:
- affirming belief in the importance of good mathematics teaching

for all pupils
- providing support and facilities for teachers in post.
- recognising that whatever the overall teacher requirement there remains a need for many more good mathematics teachers [640]. Perhaps central and local government should try a direct approach to undergraduates [641].

A list of aims for Mathematics degree courses form part of a recommendation that Universities should review their courses to make them more suited to potential teachers; e.g. by including appropriate option courses [644]. They should also and aim for broader, more flexible courses to suit single subject A level students [646]. They also believe that if more girls were encouraged to take A-level and so degrees in mathematics, the supply of teachers of mathematics would be improved [647].

The Committee welcome the Manpower Services Commission scheme for training and retraining to teach mathematics, expecially the extension to provide finance for one term and one year in-service courses for mathematics teachers.

The Committee welcome the Pilot Scheme to provide national scholarships for intending mathematics teachers [652], and suggest a scheme of flat-rate payments to suitable graduates on PGCE courses and fourth year BEd honours students tied to an undertaking to teach mathematics for a specific period [653].

To alleviate such an acute shortage as they describe, additional funding is necessary [657]. This could be used to:
- Make greater use of and increase the existing flexibility in awarding scale posts [658].
- Give LEAs discretionary power to appoint and promote competent mathematics specialists to incremental points above those defined nationally, and provide extra payments above existing scale maxima, (though these benefits might be tied to a particular post and school)[659-60]. Similar incremental discretion could be applied to competent primary mathematics co-ordinators [662].
"Any who condemn our suggestions should suggest alternative remedies to the continuing shortage of well qualified mathematics teachers" [664].

There should be a scheme to guarantee a post in a school to newly trained mathematics teachers and "the need for mathematics to be taught by well qualified teachers should take priority over the need to re-deploy staff" [665].

CHAPTER 14 Initial training courses.

Initial Training Courses are mainly Batchelor of Education degrees, available at 79 higher education establishments (including 23 polytechnics) and Postgraduate Certificate in Education (one year) courses, offered at 89 establishments (including 31 universities). These now have a higher academic level of entry, namely 2 A-levels, with O-levels (at grades A, B or C) in English and Mathematics. The Committee support the decision to require O-level mathematics, while recognising that this sometimes offers little indication of a students' potential ability to teach the subject (666-9).

This particularly affects potential primary and middle school teachers, most of whom will not study mathematics as a main subject and some of whom will not be looking forward to teaching mathematics for about 1/5 of the time (679). It is a difficult, but major, task to develop confidence and positive attitudes in such fearful students while attempting to consolidate and deepen their knowledge of mathematics, which may include filling some gaps. They see the long term solution as lying in a longer PGCE course for intending primary teachers and consider that, meanwhile, mathematically weak graduates should not be offered places on PGCE primary or middle school courses (680).

A survey of about 200 recently trained secondary teachers of mathematics raised several issues, among which was the wish for more emphasis on practical elements of the course, with compensatory reduction in time given to education theory. The Committee does not see it to be the task of ITC to prepare for the teaching of mathematics to very slow learners. This should be undertaken with experienced teachers through inservice courses.

Curriculum and method courses should stress the place of oral work, discussion and practical work while emphasising the relationship between mathematics and other areas of the curriculum.

Considering preparation for the specialist teaching of mathematics they found that by no means all those taking first method mathematics in PGCE's are mathematically well qualified, probably because mathematics posts are easier to obtain and such a choice is a useful 'insurance policy' (681-2). They believe that holding a degree with a substantial mathematical component should be necessary to get on to such courses (683). For similar reasons, mathematics is currently popular as a second method choice and it is highly probable that many will later be required to teach it. So such courses should be substantial and only open

to those whose degree contained a reasonable mathematical component or who are mathematically qualified in some other way [684].

They recommend the removal of the exemption of newly qualified mathematics graduates from the requirement to undertake initial training as a teacher. They hope that their incentives to undertake training (see chapter 13) would accompany the move [686].

Both CNAA and HMI share the view that 90 hours, suitably deployed, is a reasonable minimum for mathematics curriculum courses on the BEd and the Committee express concern about evidence that this provision is not always given. There is similar concern about the proportion of available time given to method courses on some PGCE courses and they recommend a review of the great variety of provision which exists [689]. "Proper professional preparation is of the greatest importance for intending teachers and should be given appropriate status" [700].

Turning to school experience on ITC they see the need for staff of schools and courses to come together in a well-defined and mutually supportive partnership in which they are clear about their respective responsibilities in regard to the assessment of students [690]. Where there is a conflict of aims steps should be taken to discuss and resolve such difficulties [691]. Cost restriction should not be the deciding factor in the choice of schools used for teaching practice, nor should this affect the visits from tutors [692].

Second method secondary mathematics students need opportunity to teach mathematics on teaching practice and this needs adequate supervision. In such cases, the support given by the head of mathematics will be especially important [694].

Main mathematics courses in BEd

"Teachers of mathematics must be able to take a wide view of the mathematics curriculum and be able to make informed judgements about the priorities within it". A major consideration in the validation of such a course should be whether it aims at achievement over a broad front rather than seeks to achieve depth by restricting its coverage to only a few areas of mathematics [695].

In such courses groups should not be so large as to discourage discussion and co-operative working, but should be large enough to permit a sufficient interplay of ideas. If a course regularly recruits less than eight students its continued existence should be called into question [698].

Induction

"There is no major profession to which a new entrant, however thorough his initial training can be expected immediately to make a full contribution". Induction should be "systematic programme of professional initiation, guided experience and further study" (quoted from "Education: A Framework for Expansion" HMSO 1972) [702]. This intention, of a decade ago has not yet been realised, though many ITC's have been structured on the assumption that it would happen. They believe that a proper programme of induction is necessary in the first year, which should include both a lighter teaching load and support from all possible agencies in and out of school [705].

They recommend that HMI should continue to be involved with teacher training courses and should be given the duty of appraising the ITC which university departments of education provide [710].

CHAPTER 15 In-Service Support for Teachers

A special case is made for the in-service support of the mathematics teacher [715-8].

The report considers that co-operative working with a school or department is both possible and effective. They agree with Richardson that "every school has within it the ingredients of a kind of educational workshop" and can be helped in this respect by the support of various outside agencies [720].

Local teachers centres and mathematics centres play an important part in providing facilities for teachers to meet and have a valuable part to play in improving mathematics teaching. Local teachers centres that have been closed should be re-opened and at least one adequately resourced mathematics centre provided by each L.E.A. [725-7]. Arrangements should also be made to enable teachers to visit each others schools.

Every effort should be made to encourage teachers to join professional mathematical education associations and the associations themselves should do as much as possible to develop local activities [730]. The committee consider that "it is vital that (the associations) should continue to be able to present informed and independent opinions", and that LEAs should support teachers involved with these associations' working groups and committees [731].

The advisory staff of an LEA have an essential role to play in in-service training and need to be in regular contact with a number of relevant agencies [732]. LEAs need to provide an adequate advisory service for its mathematics teachers [734]. The appointment of mathematics advisers should be followed by a programme of induction and further training should be provided

from time to time [736-8]. Advisory mathematics teachers where they exist can provide valuable help in school-based in-service [739].

Establishments of higher education have made a major contribution to in-service support of teachers. These institutions can provide resources and support for teachers across LEA boundaries. The committee considers that "school - teacher fellowships" are of great value, although very few exist. They report that some of those working in universities feel that time spent on in-service work is detrimental to their academic careers since those responsible for making more senior appointments value publication for academic research more highly [742]. The committee regret both this and the fact that unfavourable methods of 'costing' the beneficial consultancy work carried out in schools by some training institutes can limit the time they feel able to spend on it. In-service courses are not the most important part of in-service work, but have an essential role to play. Courses held during school hours are considered to have advantages [744-5]. Full time courses allow for deeper reflection and discussion [746]. Efforts should also be made to arrange follow up sessions to all forms of courses [748]. The committee regrets that there are unhelpful limits on financial re-imbursements for teachers undertaking in-service work [763] and also recommends that the number of teachers seconded to full time courses is substantially increased [749]. They also feel that if every teacher were required by his conditions of service to spend a certain number of days each year in in-service education, the recommendations made throughout the report could be better implemented [759].

The Mathematics Association's Diploma in Mathematical Education is seen as making a substantial and effective contribution and deserves every support. The Open University in-service courses could also play a major part in the future, if financially supported [750-3]. The committee would welcome the establishment of a few more Centres for Mathematical Education based on existing institutes known to be strong in the field of mathematical education with a strongly classroom related bias [758]. Disincentives to involvement in in-service should be removed where they currently exist [759-63]. The Secretaries of State have the powers to provide funds for programmes of in-service. These must be put into operation to provide sufficient resources for additional support, without which the quality of mathematics teaching is considered unlikely to improve [765-6].

CHAPTER 16 Some other matters
This chapter deals with four disconnected particular issues which have not fitted elsewhere [767]

Mathematics in Middle Schools [768-73]
The mixed character of the relation of 8-12 or 9-13 middle schools to the primary/secondary system, and the training and supply of teachers leads to worrying problems. These can be manifest in too high a proportion of mathematics teaching of the older pupils being carried out by teachers with limited mathematical qualifications. This is the case whether the background is primary or, as is often the case, where the middle school was formed from a secondary modern school. Concern was expressed that mathematics co-ordinators were not all well qualified and that they did not necessarily have time to exercise practical co-ordinating responsibilities through working closely with teacher colleagues and liaising with other schools.

The teaching of statistics
The Committee draws attention to the newly developed characteristic of secondary school syllabuses in connection with the frequent failure to appreciate the way statistics provides an approach to judgements in the face of uncertainty. The theoretical way in which the mathematics syllabus often treats the subject can be limiting. There is a need for further materials to encourage a practical approach.

Mathematical education in other countries [782-792].
Useful comments are made about the experience of visiting other countries and discussing diverse practices. A chief consequence was the recommendation that there should be an investigation into how a requirement for suitable academic qualifications should be introduced over a period of years, and that first steps be taken soon.

Statistical information relating to Mathematical Education [793-7]
The committee makes recommendations about regular collection of certain data which it has had to arrange for itself or which is unobtainable. It also suggests that the first subject identified for a BEd degree should not be education but should be the subject of the main course taken. The staffing survey of 1977 should be repeated every five years.

CHAPTER 17 The Way Ahead
This chapter contains issues considered of major importance. A summary does not do the chapter justice as it purveys as a whole the balanced nature of the work of the Committee [800].
Sympathy is expressed for the many teachers who have given

willingly of their time to improve standards, in an increasingly critical atmosphere. Nevertheless the mathematical education which many pupils are receiving is not satisfactory. "We therefore believe that major changes are essential" [800].
A differentiated curriculum must be provided to enable pupils to develop their diverse capacities to the full. Examinations at secondary level must also be better suited to the needs of pupils than is the case at present [801].
Paragraphs [803-8] recommend active co-operation of the six main agencies : Teachers at the front, LEAs providing support, examining boards, central government, training institutions, and those funding and engaged on curriculum development and educational research.

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2 "Mathematics in Employment 16-18". Available from: W J Vennard, School of Mathematics, University of Bath Bath, BA2 7AY. Price £4 inc p&p.

3 Reports on the industry surveys carried out at the University of Nottingham. Available from: Shell Centre for Mathematical Education, University of Nottingham, Nottingham, NG7 2RD.

4 "A review of research in Mathematical Education: introduction and summary". Available from the same address as 3.

5 1977 Secondary School Staffing Survey. HMSO

6 75% in 1938; 30% in 1964; 10% in 1979

7 "The Teacher, the School and the Task of Management" Elizabeth Richardson, published by Heinemann Educational 1973. Price £2.75

Other Publications Available From ATM Include:

Title	Price
Notes on Mathematics in Primary Schools	£3.60
Mathematical Activities from Poland - a new primary pamphlet	£1.50
A Boolean Anthology	£1.00
Mathematics for Sixth Formers	£1.50
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